MATHEMATICS

(Two hours and a half)

Specimen paper (solved)

*Answers to this Paper must be written on the paper provided separately. You will **not** be allowed to write during the first **15** minutes.*

This time is to be spent in reading the question paper.

The time given at the head of this Paper is the time allowed for writing the answers.

---

*Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*

All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.

Omission of essential working will result in loss of marks.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables are provided.

SECTION A (40 Marks)

*(Attempt **all** questions from this Section)*

Question 1

(a) Find the value of 'k' if $4x^3 - 2x^2 + kx + 5$ leaves remainder -10 when divided by $2x + 1$. [3]

Solution.

$f(x) = 4x^3 - 2x^2 + kx + 5$ .... (i)

$2x + 1 = 0 \quad \Rightarrow \quad x = -\frac{1}{2} - \frac{1}{2}$

Now, put $x = -\frac{1}{2}$

$$f\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^3 - 2\left(-\frac{1}{2}\right)^2 + k\left(-\frac{1}{2}\right) + 5$$

$$f\left(-\frac{1}{2}\right) = -4\times\frac{1}{8} - 2\times\frac{1}{4} - k\times\frac{1}{2} + 5$$

$$f\left(-\frac{1}{2}\right) = -\frac{1}{2} - \frac{1}{2} - \frac{k}{2} + 5 = \frac{-1-1-k+10}{2}$$

$$= \frac{8-k}{2}$$

According to problem,

$$f\left(-\frac{1}{2}\right) = -10$$

$\Rightarrow \quad \frac{8-k}{2} = -10$

$\Rightarrow \quad 8 - k = -2 \times 10 \qquad \Rightarrow \quad 8 + 20 = k$

$\therefore$ the value of k = 28 (Ans)

(b) Amit deposits Rs 1600 per month in a bank for 18 months in a recurring deposit account. If he gets Rs. 31,080 at the time of maturity, what is the rate of interest per annum? [3]

Solution.

Monthly amount (P) = Rs. 1600

month (n) = 18

Maturity value (M.V) = 31,080

We know,

MV = n x P + I

31,080 = 18 x 1600 + I

I = 31080 - 18 x 1600 = 2280

Now,

$$\text{Interest (I)} = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$2280 = 1600 \times \frac{18 \times 19}{2 \times 12} \times \frac{r}{100}$$

$$r = \frac{2280 \times 2 \times 12 \times 100}{1600 \times 18 \times 19} = 10\%$$

$\therefore$ the rate of interest is 10% (Ans)

(c) A shopkeeper bought an article with market price ₹ 1200 from the wholesaler at a discount of 10%. The shopkeeper sells this article to the customer on the market price printed on it. If the rate of GST is 6%, then find:

(i) GST paid by the wholesaler.

(ii) Amount paid by the customer to buy the item. [4]

Solution.

<u>Wholesaler</u>

Market price = ₹ 1200

Discount = 10%

Discounted price or selling price

= 1200 - 1200 x 10%

= 1200 - 120 = ₹ 1080

(i) GST paid by wholesaler = SP x rate of GST

= 1080 x 6% = ₹ 64.80. (Ans)

Shopkeeper

Market price = ₹ 1200

Selling price = ₹ 1200

GST collected by shopkeeper = 1200 x 6% = ₹ 72

(ii) Amount of bill paid by the customer = SP + GST

= 1200 + 72

= ₹ 1272. (Ans)

Question 2.

(a) Solve the following inequation and represent your solution on the real number line:

$-5\frac{1}{2} - x \leq \frac{1}{2} - 3x \leq 3\frac{1}{2} - x$, $x \in R$ [3]

Solution.

$-5\frac{1}{2} - x \leq \frac{1}{2} - 3x$ ....(i) and $\frac{1}{2} - 3x \leq 3\frac{1}{2} - x$ ....(ii)

From eq (i)

$-\frac{11}{2} - x \leq \frac{1}{2} - 3x$

or, $-\frac{11}{2} - \frac{1}{2} \leq -3x + x$

or, $-6 \leq -2x$ or, $x \leq 3$

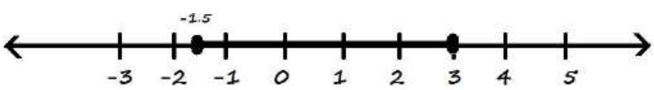
From eq (ii)

$\frac{1}{2} - 3x \leq \frac{7}{2} - x$

or, $\frac{1}{2} - 3x \leq \frac{7}{2} - x$

or, $\frac{1}{2} - \frac{7}{2} \leq -3x - x$ or, $x \geq -1.5$

Solution set : $\{x : -1.5 \leq x \leq 3, x \in R\}$



(b) Find the 16th term of the A.P. 7, 11, 15, 19.... Find the sum of the first 6 terms. [3]

Solution.

Series : 7, 11, 15, 19....

First term (a) = 7 and common difference (d) = 11- 7 = 4

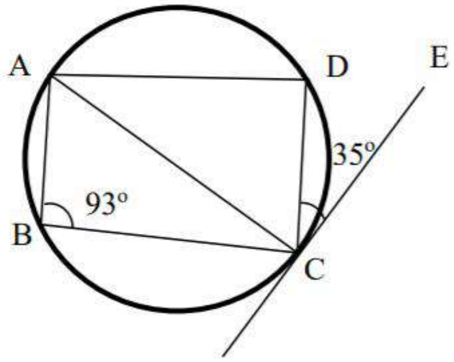
Now, 16^{th} term = 7 + (16 - 1) x 4 [$\because$ $t_n = a + (n - 1)d$]

= 67

Sum of first 6 terms = $\frac{6}{2}[2\times7+(6-1)\times4]$ $\left[\because S_n = \frac{n}{2}[2a+(n-1)d]\right]$

= 3(14 + 20) = 102.

(c) In the given figure CE is a tangent to the circle at point C. ABCD is a cyclic quadrilateral. If $\angle ABC = 93^o$ and $\angle DCE = 35^o$



Find: (i) $\angle ADC$ (ii) $\angle CAD$ (ii) $\angle ACD$ [4]

Solution.

According to the question,

(i) As, $ABCD$ is a cyclic quadrilateral.

$\therefore$ $\angle ABC + \angle ADC = 180°$

[$\therefore$ sum of opposite angles of a cyclic quadrilateral is 180°]

$\Rightarrow$ $93° + \angle ADC = 180°$ [$\because \angle ABC = 93°$]

$\Rightarrow$ $\angle ADC = 180° - 93° = 87°$

(ii) $\angle CAD = \angle DCE$

[by alternate segment theorem]

$\Rightarrow$ $\angle CAD = 35°$ [$\because \angle DCE = 35°$]

(iii) In ΔACD,

$\angle ACD + \angle ADC + \angle CAD = 180°$

[angle sum property of triangle]

$\Rightarrow$ $\angle ACD + 87° + 35° = 180°$

$\Rightarrow$ $\angle ACD = 180° - (87° + 35°)$

$= 180° - 122° = 58°$

Question 3.

(a) Prove the following identity : [3]

$$\frac{\sec A}{\sec A-1}+\frac{\sec A}{\sec A+1}=2\cos ec^2 A$$

Solution.

L.H.S : $\frac{\sec A}{\sec A-1}+\frac{\sec A}{\sec A+1}$

$$= \frac{\sec A(\sec A+1)+\sec A(\sec A-1)}{(\sec A-1)(\sec A+1)}$$

$$= \frac{\sec^2 A+\sec A+\sec^2 A-\sec A}{\sec^2 A-1}$$

$$= \frac{2\sec^2 A}{\tan^2 A} = 2\times\frac{1}{\cos^2 A}\times\frac{\cos^2 A}{\sin^2 A} = \frac{2}{\sin^2 A}$$ = 2cosec² A = **RHS** (Proved)

(b) Find x and y if : [3]

$$3\begin{bmatrix}5 & -6\\ 4 & x\end{bmatrix}-\begin{bmatrix}6 & y\\ 0 & 6\end{bmatrix}=3\begin{bmatrix}3 & -2\\ 4 & 0\end{bmatrix}$$

Solution.

$$3\begin{bmatrix}5 & -6\\ 4 & x\end{bmatrix}-\begin{bmatrix}6 & y\\ 0 & 6\end{bmatrix}=3\begin{bmatrix}3 & -2\\ 4 & 0\end{bmatrix}$$

or, $\begin{bmatrix}15 & -18\\ 12 & 3x\end{bmatrix}-\begin{bmatrix}6 & y\\ 0 & 6\end{bmatrix}=\begin{bmatrix}9 & -6\\ 12 & 0\end{bmatrix}$

or, $\begin{bmatrix}15-6 & -18-y\\ 12 & 3x-6\end{bmatrix}=\begin{bmatrix}9 & -6\\ 12 & 0\end{bmatrix}$

By the equality of two matrices we have,

$-18 - y = -6$ or, $y = -12$

And $3x - 6 = 0$ or, $x = 2$

∴ $x = 2$ and $y = -12$ (Ans)

(c) For what value of 'k' will the following quadratic equation:

$(k + 1)x^2 - 4kx + 9 = 0$ have real and equal and equal roots? Solve the equation. [4]

Solution.

We have, $a = (k + 1)$, $b = -4k$ and $c = 9$

For real and equal roots,

$b^2 - 4ac = 0$

or, $(-4k)^2 - 4(k + 1) \times 9 = 0$

or, $16k^2 - 36(k + 1) = 0$

or, $16k^2 - 36k - 36 = 0$

or, $4(4k^2 - 9k - 9) = 0$

or, $4k^2 - (12 - 3)k - 9 = 0$

or, $4k^2 - 12k + 3k - 9 = 0$ $(4k + 3)(k - 3) = 0$ $k = 3$ and $-\frac{3}{4}$

For $k = -\frac{3}{4}$

Now, if k = 3 equation becomes

$4x^2 - 12x + 9 = 0$

Or, $4x^2 - 6x - 6x + 9 = 0$

Or, $2x(2x - 3) - 3(2x - 3) = 0$

Or, $(2x - 3)(2x - 3) = 0$

Or, $x = \frac{3}{2}, \frac{3}{2}$

Again, if $k = -\frac{3}{4}$, then equation becomes

Or, $\frac{1}{4}x^2 + 3x + 9 = 0$

Or, $x^2 + 12x + 36 = 0$

Or, $(x + 6)^2 = 0$

Or, $x = -6, -6$

Question 4.

(a) A box consists of 4 red, 5 black and 6 white balls. One ball is drawn out at random. Find the probability that the ball drawn is : [3]

(i) Black (ii) red or white

Solution.

We have 4 red, 5 black and 6 white balls.

$\therefore$ total number of balls = 4 + 5 + 6 = 15

(i) $P(\text{Black}) = \frac{\text{number of black balls}}{\text{total number of balls}} = \frac{5}{15} = \frac{1}{3}$

(ii) P (Red or White)

$= P(\text{Red}) + P(\text{White})$

$= \frac{\text{number of red balls}}{\text{total number of balls}} + \frac{\text{number of white balls}}{\text{total number of balls}}$

$$= \frac{4}{15} + \frac{6}{15} = \frac{10}{15} = \frac{2}{3}$$

(b) Calculate the median and mode for the following distribution. [3]

Weight (in kg)	35	47	52	56	60
Number of students	4	3	5	3	2

Solution.

Median cumulative frequency table for given distribution is given below :

x_i	f_i	cf
35	4	4
47	3	7
52	5	12
56	3	15
60	2	17

Here, N = 17 (odd)

$\therefore$ Median = $\left[\frac{1}{2}(N+1)\right]$th value $\Rightarrow \left[\frac{1}{2}(17+1)\right] = 9$th value

$\therefore$ Median = 52

Mode is the data having highest frequency. So, 52 is mode as it has highest frequency i.e., 5 among all these data.

(c) A solid cylinder of radius 7 cm and height 14 cm is melted and recast into solid spheres each of radius 3.5 cm. Find the number of spheres formed. [4]

Solution.

For cylinder ,

Radius (R_1) = 7 cm, height (H_1) = 14 cm

$\therefore$ Volume of cylinder = $\pi R_1^2 H_1 = \pi(7)^2 \times 14$

For sphere , Radius (R_2) = 3.5 cm

$\therefore$ volume of sphere = $\frac{4}{3}\pi R_2^2 = \frac{4}{3}\pi(35)^2$

Let n sphere be formed .

n × volume of sphere = volume of cylinder

$\Rightarrow n \times \frac{4}{3}\pi(35)^2 = \pi(7)^2 \times 14$

$$\Rightarrow n = \frac{7 \times 7 \times 14 \times 3}{4 \times 35 \times 35 \times 35} = 12$$

Hence, the number of spheres so formed is 12.

Section - B

*(Attempt **any four** questions from this section)*

Question 5.

(a) The 2nd and 45th term of an arithmetic progression are 10 and 96 respectively. Find and first term and the common difference and hence find the sum of first 15 terms. [3]

Solution.

Let the first term be a and common difference be d of the AP.

Now, $a_2 = 10 \Rightarrow a + d = 10$

And $a_{45} = 96 \Rightarrow a + 44d = 96$ $[\because a_n = a + (n - 1)d]$

From eq (i), put the value of a = 10 - d in eq (ii), we get

$(10 - d) + 44d = 96 \Rightarrow 43d = 86 \Rightarrow d = 2$

$a = 10 - d = 10 - 2 = 8$

Hence, a = 8 and d = 2 .

Again, we know that,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{15} = \frac{15}{2}[2 \times 8 + (15-1) \times 2]$$

$$= \frac{15}{2}[16 + 28] = \frac{15 \times 44}{2}$$

$$= 15 \times 22 = 330$$

Hence, the sum of the first 15 terms is 330

(b) If $A = \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}$, find the matrix B such that $A^2 - 2B = 3A - 5I$, where I is 2×2 identity matrix. [3]

Solution.

We have,

$$A = \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \quad A^2 = A \cdot A = \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times3+(-1)\times0 & 3\times(-1)+(-1)\times2 \\ 0\times3+2\times0 & 0\times(-1)+2\times2 \end{bmatrix}$$

$$\therefore \quad A^2 = \begin{bmatrix} 9 & -5 \\ 0 & 4 \end{bmatrix}$$

Now, we have

$$A^2 - 2B = 3A - 5I \Rightarrow 2B = A^2 - 3A + 5I$$

$$\Rightarrow \quad 2B = \begin{bmatrix} 9 & -5 \\ 0 & 4 \end{bmatrix} - 3\begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad 2B = \begin{bmatrix} 9 & -5 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -9 & 3 \\ 0 & -6 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$\Rightarrow \quad 2B = \begin{bmatrix} 9-9+5 & -5+3+0 \\ 0+0+0 & 4-6+5 \end{bmatrix}$$

$$\Rightarrow \quad 2B = \begin{bmatrix} 5 & -2 \\ 0 & 3 \end{bmatrix}$$

$$\therefore \quad B = \frac{1}{2}\begin{bmatrix} 5 & -2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 5/2 & -1 \\ 0 & 3/2 \end{bmatrix}$$

(c) With the help of a graph paper, taking 1cm =1unit along both X and Y-axis:

(i) Plot points A (0, 3), B (2, 3), C (3, 0), D (2, -3), E (0, -3)

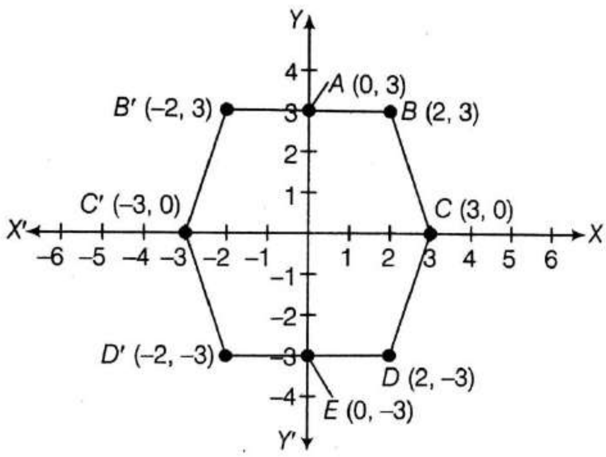
(ii) Reflect points B, C and D on the y axis and name them as B', C' and D' respectively.

(iii) Write the co-ordinates of B', C' and D'.

(iv) Write the equation of line B' D'.

(v) Name the figure BCDD'C'B'B. [4]

Solution.



(i) See the graph.

(ii) See the graph.

(iii) $B' = (-2, 3), C' = (-3, 0)$ and $D' = (-2, -3)$

(iv) Equation of line passing through $B'(-2, 3)$ and $D'(-2, -3)$ is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - 3 = \frac{-3-3}{-2-(-2)}(x - (-2))$$

$$\Rightarrow \quad y - 3 = \frac{-6}{0}(x + 2) \Rightarrow x + 2 = 0$$

(v) From the graph it is clear that $BCDD'C'B'$ is hexagon.

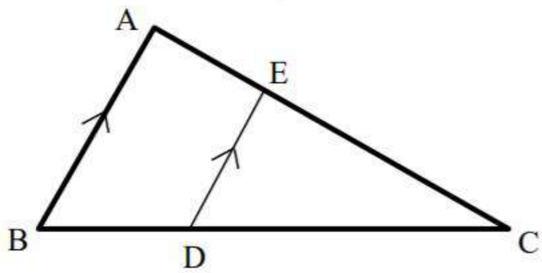
Question 6.

(a) In Δ ABC and ΔEDC, AB is parallel to ED.BD = $\frac{1}{3}$ BC and AB = 12.3 cm. [3]

(i) Prove that ΔABC ~ΔEDC.

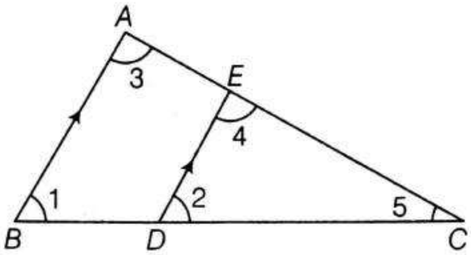
(ii) Find DE

(iii) Find: $\frac{\text{area of } \Delta EDC}{\text{area of } \Delta ABC}$



Solution.

According to the question,



(i) Since, $AB \parallel DE$

$\therefore \angle 1 = \angle 2$ and $\angle 3 = \angle 4$

[∵ corresponding angles]

Now, in ΔABC and ΔEDC

$\angle 1 = \angle 2$ [proved above]

$\angle 3 = \angle 4$ [proved above]

$\angle 5 = \angle 5$ [common]

$\therefore$ By AAA criteria, $\Delta ABC \sim \Delta EDC$

(ii) Since, $\Delta ABC \sim \Delta EDC$

$$\therefore \frac{AB}{ED} = \frac{BC}{DC} \Rightarrow \frac{AB}{ED} = \frac{BC}{BC - DB}$$

$$\Rightarrow \frac{AB}{ED} = \frac{BC}{BC - \frac{1}{3}BC} \quad \left[\because BD = \frac{1}{3}BC\right]$$

$$\Rightarrow \frac{AB}{ED} = \frac{BC}{\frac{2}{3}BC} \Rightarrow \frac{AB}{ED} = \frac{3}{2}$$

$$\Rightarrow ED = \frac{2}{3}AB = \frac{2}{3} \times 12.3 = 8.2 \text{ cm}$$

[$\because AB = 12.3$ cm]

(iii) We know that, if two triangles are similar then the ratio of their areas is equal to the square of the ratio of their corresponding sides.

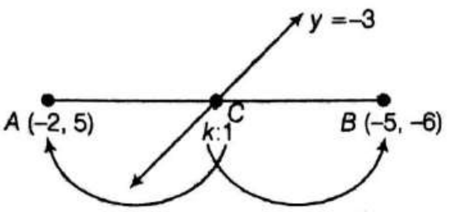
$$\therefore \frac{\text{area of } \Delta EDC}{\text{area of } \Delta ABC} = \left(\frac{ED}{AB}\right)^2 = \left(\frac{8.2}{12.3}\right)^2$$

$$= \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

(b) Find the ratio in which the line joining (-2, 5) and (-5, -6) is divided by the line y = -3. Hence find the point of intersection. [3]

Solution.

Let the line y = -3 divides te line segment joining A(-2,5) and B(-5,-6) in the ratio k:1 at the point C.



Then, the coordinates of $C = \left(\frac{-5k-2}{k+1}, \frac{-6k+5}{k+1}\right)$

But C lies on $y = -3$, therefore

$$\frac{-6k+5}{k+1} = -3$$

$\Rightarrow \quad -6k+5=-3k-3$

$\Rightarrow \quad -6k+3k=-3-5$

$\Rightarrow \quad -3k=-8$

$\Rightarrow \quad k=\frac{8}{3}$

Hence, the required ratio is 8 : 3 internally.

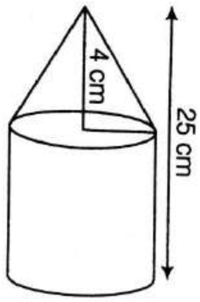
On putting $k=\frac{8}{3}$ in the coordinates of C, we get

$$C=\left(\frac{-5\times\frac{8}{3}-2}{\frac{8}{3}+1}, \frac{-6\times\frac{8}{3}+5}{\frac{8}{3}+1}\right)$$

$$=\left(\frac{-40-6}{8+3}, \frac{-48+15}{8+3}\right)$$

$$=\left(-\frac{46}{11}, \frac{-33}{11}\right)=\left(\frac{-46}{11}, -3\right)$$

(c) The given solid figure is a cylinder surmounted by a cone. The diameter of the base of the cylinder is 6 cm. The height of the cone is 4 cm and the total height of the solid is 25 cm. Take $\pi = \frac{22}{7}$. [4]



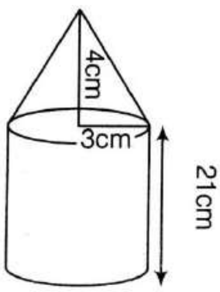
Find the:

(i) Volume of the solid

(ii) Curved surface area of the solid Give your answers correct to the nearest whole number

Solution.

For cylinder,
Radius $(R_1)=3$ cm,
Height $(H_1)=25-4=21$ cm
for cone,
Radius $(R_2)=3$ cm,
Height $(H_2)=4$ cm



(i) Volume of the solid = Volume of cylinder + Volume of cone

$$= \pi R_1^2 H_1 + \frac{1}{3}\pi R_2^2 H_2$$

$$= \pi (3)^2 \times 21 + \frac{1}{3}\pi (3)^2 \times 4 \quad = 189\pi + 12\pi = 201\pi$$

$$= 201 \times \frac{22}{7} = 631.714 \approx 632 \text{ cm}^3$$

(ii) Let l be the slant height of the cone.

Then, slant height,

$$l = \sqrt{R_2^2 + H_2^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16}$$

$$= \sqrt{25} = 5 \text{ cm}$$

So, curved surface area of the solid = curved surface area of cylinder + curved surface area of cone

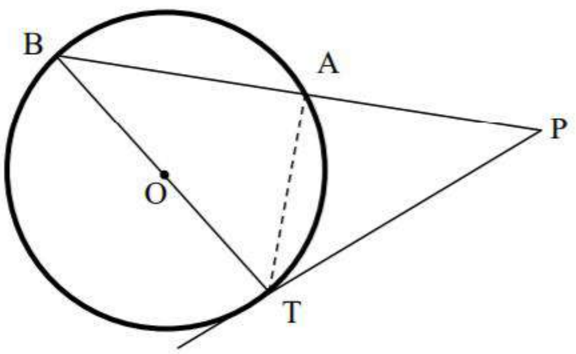
$$= 2\pi R_1 H_1 + \pi R_2 l$$

$$= 2\pi \times 3 \times 21 + \pi \times 3 \times 5 = 126\pi + 15\pi$$

$$= 141\pi = 141 \times \frac{22}{7} \approx 443 \text{ cm}^2$$

Question 7.

(a) In the given figure, PAB is a secant and PT a tangent to the circle with centre O. If $\angle ATP = 40^\circ$, PA = 9 cm and AB = 7 cm



Find:

(i) $\angle APT$

(ii) length of PT [3]

Solution.

(i) Given, $\angle ATP = 40^\circ$

Now, we know that angle in semi-circle is 90°.

$\therefore \quad \angle TAB = 90^\circ$

Again, $\angle TAB + \angle TAP = 180^\circ$ [linear pair]

$\Rightarrow \quad 90° + \angle TAP = 180°$

$\Rightarrow \quad \angle TAP = 180° - 90° = 90°$

Now, in ΔAPT,

$\angle ATP + \angle TAP + \angle APT = 180°$ [$\because$ sum of all the angles of a triangle is 180°]

$\Rightarrow \quad 40° + 90° + \angle APT = 180°$

$\Rightarrow \quad \angle APT = 180° - 130° = 50°$

(ii) We know that,

$PT^2 = PA \times PB \Rightarrow PT^2 = 9 \times 16$ [$\because PA = 9$ cm, $PB = PA + AB = 9 + 7 = 16$ cm]

$\Rightarrow \quad PT^2 = 144 \qquad PT = 12$ cm

(b) The 1st and the 8th term of a GP are 4 and 512 respectively.

Find:

(i) the common ratio

(ii) the sum of its first 5 terms. [3]

Solution.

Let a be the first term and r be the common ratio of the given GP, then

$a_n = ar^{n-1}$.

(i) Now, according to the question

first term, $a = 4$

and $a_8 = 512$

$\Rightarrow \quad ar^7 = 512$

$\Rightarrow \quad 4r^7 = 512 \qquad [\because a = 4]$

$\Rightarrow \quad r^7 = 128 \Rightarrow r^7 = 2^7 \Rightarrow r = 2$

Hence, the common ratio is 2.

(ii) We know that,

$$S_n = a\left[\frac{r^n - 1}{r - 1}\right] \qquad [\because r > 1]$$

$$\Rightarrow \quad S_5 = 4\left(\frac{2^5 - 1}{2 - 1}\right) = 4(32 - 1) = 4 \times 31 = 124$$

(c) The mean of the following distribution is 49. Find the missing frequency 'a'. [4]

Class	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
Frequency	15	20	30	a	10

Solution.

Class	x_i	Frequency (f_i)	$f_i\ x_i$
0 - 20	10	15	150
20 - 40	30	20	600
40 - 60	50	30	1500
60 - 80	70	a	$70a$
80 - 100	90	10	900
Total		$\Sigma f_i = 75 + a$	$\Sigma f_i x_i = 3150 + 70a$

$\therefore$ $$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$\therefore$ $$49 = \frac{3150 + 70a}{75 + a}$$

$\Rightarrow$ $$49 \times 75 + 49a = 3150 + 70a$$

$\Rightarrow$ $$3675 + 49a = 3150 + 70a$$

$\Rightarrow$ $$70a - 49a = 3675 - 3150$$

$\Rightarrow$ $$21a = 525$$

$\Rightarrow$ $$a = 25$$

Question 8.

(a) Prove the following identity (sinA + cosecA)^2 + (cosA + secA)^2 = 5 + sec^{2}A . cosec^{2}A [3]

Solution.

$$\text{LHS} = (\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2$$

$$= \sin^2 A + \cos ec^2 A + 2\sin A \,\text{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$$

$$[\because (a+b)^2 = a^2 + b^2 + 2ab]$$

$$= (\sin^2 A + \cos^2 A) + \left(2\sin A \times \frac{1}{\sin A}\right) + \left(2\cos A \times \frac{1}{\cos A}\right) + \sec^2 A + \text{cosec}^2 A$$

$$\left[\because \sec A = \frac{1}{\cos A}, \text{cosec } A = \frac{1}{\sin A}\right]$$

$$= 1 + 2 + 2 + \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A}$$

$$= 5 + \frac{\sin^2 A + \cos^2 A}{\cos^2 A \sin^2 A} \quad [\because \sin^2 A + \cos^2 A = 1]$$

$$= 5 + \frac{1}{\cos^2 A \sin^2 A} = 5 + \sec^2 A \,\text{cosec}^2 A \quad = \text{RHS}$$

(b) Find the equation of the perpendicular bisector of line segment joining A(4, 2) and B(-3, -5). [3]

Solution.

We have, $A(4, 2)$ and $B(-3, -5)$.

Mid-point of the line segment joining the points A and B

$$=\left(\frac{4-3}{2}, \frac{2-5}{2}\right)=\left(\frac{1}{2}, \frac{-3}{2}\right) \qquad \left[\because \text{mid-point}=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$$

Slope of the line joining points $A(4, 2)$ and $B(-3, -5)$

$$=\frac{-5-2}{-3-4}=\frac{-7}{-7}=1 \qquad \left[\because \text{slope}=\frac{y_2-y_1}{x_2-x_1}\right]$$

Since, the required line is perpendicular to the AB, slope of the required line is $\frac{1}{-1}=-1.$

We have the equation of a line passing through (x_0, y_0) and slope m is $y-y_0=m(x-x_0).$

Since, the required line passes through point $\left(\frac{1}{2}, \frac{-3}{2}\right)$ and having slope -1.

$\therefore$ Equation of the required line is $y+\frac{3}{2}=-1\left(x-\frac{1}{2}\right)$

$$\Rightarrow \quad 2y+3=-2x+1 \Rightarrow 2x+2y+2=0$$

(c) Using properties of proportion, find x : y if $\frac{x^3+12x}{6x^2+8}=\frac{y^3+27y}{9y^2+27}$ [4]

Solution.

We have, $\frac{x^3+12x}{6x^2+8}=\frac{y^3+27y}{9y^2+27}$

Using componendo and dividendo, we get

$$\frac{x^3+12x+6x^2+8}{x^3+12x-6x^2-8}=\frac{y^3+27y+9y^2+27}{y^3+27y-9y^2-27}$$

$$\Rightarrow \frac{(x+2)^3}{(x-2)^3}=\frac{(y+3)^3}{(y-3)^3} \Rightarrow \frac{x+2}{x-2}=\frac{y+3}{y-3}$$

Using componendo and dividendo again, we get

$$\frac{(x+2)+(x-2)}{(x+2)-(x-2)}=\frac{(y+3)+(y-3)}{(y+3)-(y-3)}$$

$$\Rightarrow \frac{2x}{4}=\frac{2y}{6} \Rightarrow \frac{x}{2}=\frac{y}{3} \Rightarrow \frac{x}{y}=\frac{2}{3}$$

$$x:y=2:3$$

Question 9.

(a) The difference of the squares of two natural numbers is 84. The square of the larger number is 25 times the smaller number. Find the numbers. [4]

Solution.

Let the two numbers be x and y, $(x > y)$. Now, according to the question.

$$x^2 - y^2 = 84 \quad \ldots(i)$$

and $$x^2 = 25y \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$25y - y^2 = 84$$

$$\Rightarrow \quad y^2 - 25y + 84 = 0$$

$$\Rightarrow \quad y^2 - 4y - 21y + 84 = 0$$

$$\Rightarrow \quad y(y-4) - 21(y-4) = 0$$

$$\Rightarrow \quad (y-21)(y-4) = 0$$

$$\Rightarrow \quad y = 21, 4$$

When $y = 21$,

then $x^2 = 25 \times 21 \Rightarrow x = \pm 5\sqrt{21}$

which is not possible as x is natural number.

When $y = 4$,

then $$x^2 = 25 \times 4$$

$$x^2 = 100$$

$$\Rightarrow \quad x = \pm 10$$

$$\Rightarrow \quad x = 10 \quad [\because x \text{ is a natural number}]$$

Hence, the required numbers are 10 and 4.

(b) The following table shows the distribution of marks in Mathematics :

Marks (less than)	No. of students
10	7
20	28
30	54
40	71
50	84
60	105
70	147
80	180

With the help of a graph paper, taking 2 cm = 10 units along one axis and 2 cm = 20 units along the other axis, plot an ogive for the above distribution and use it to find the:

(i) median.

(ii) number of students who scored distinction marks (75% and above)

(iii) number of students, who passed the examination if pass marks is 35%. [6]

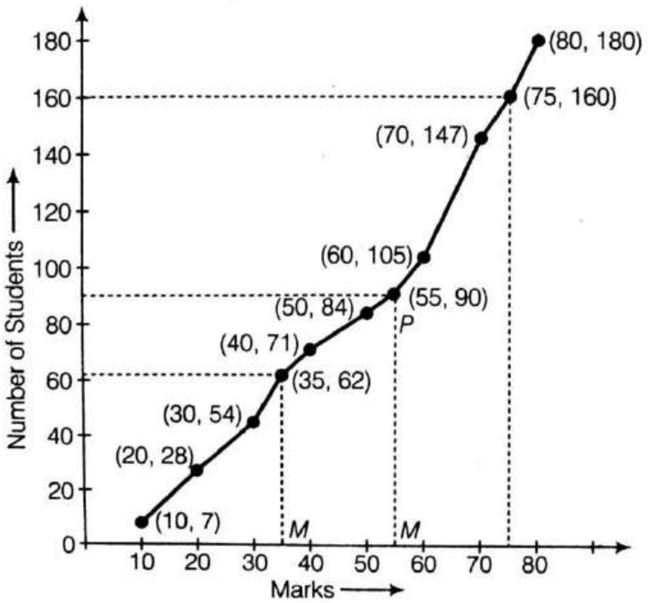
Solution.

Given table is shown below

Marks (Less than)	Number of Students
10	7
20	28
30	54
40	71
50	84
60	105
70	147
80	180

Now, we take marks on X-axis and number of students on Y-axis.

Plot the points (10, 7), (20, 28), (30, 54), (40, 71), (50, 84), (60, 105), (70, 147) and (80, 180).



(i) Here, $N = 180$

$\therefore \quad \frac{N}{2} = 90°$

In order to find the median let us first locate the point corresponding to 90 on Y-axis.

and from-this draw a horizontal line to meet the ogive at P. Now, through P, draw a vertical line to meet X-axis at M. The x-coordinate of M is 55.

$\therefore$ Median from the graph $=55$

(ii) Number of students who scored distinction marks (75% and above)

$$=180-160=20$$

(iii) Number of students whose passed $=180-62=118$

Question 10.

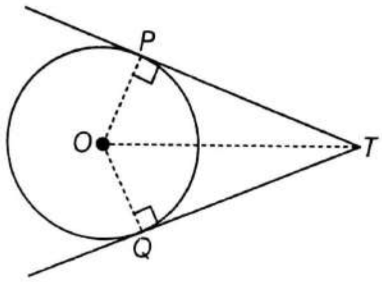
(a) Prove that two tangents drawn from an external point to a circle are of equal length.[3]

Solution.

Given, two tangents TP and TQ are drawn from a point T to a circle with centre O.

To prove $TP=TQ$

Construction Join OP, OQ and OT



Proof

Consider the triangles OPT and OQT.

$OP=OQ$ [radii]

$\angle OPT=\angle OQT=90°$

[$\because$ radius is perpendicular to the tangent at the point of contact]

$OT=OT$ [common]

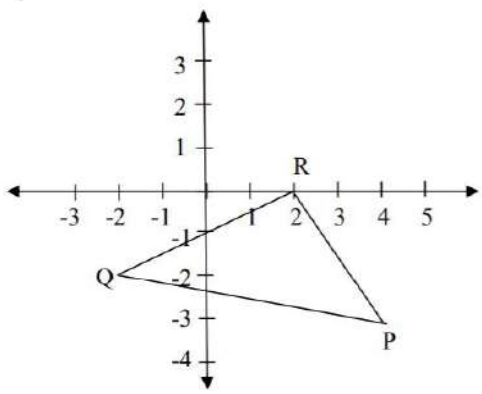
$\therefore$ By RHS criteria, $\Delta OPT \cong \Delta OQT$

$\therefore$ By $CPCT$, $TP=TQ$ **Hence proved.**

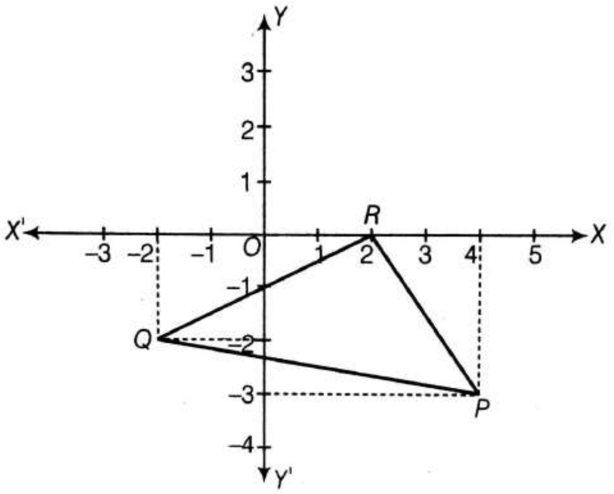
(b) From the given figure find the:

(i) Coordinates of points P, Q, R.

(ii) Equation of the line through P and parallel to QR. [3]



Solution.



(i) From the graph it is clear that, $P(4,-3)$, $Q(-2,-2)$ and $R(2,0)$

(ii) Slope of line joining $QR = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-2)}{2-(-2)} = \frac{2}{4} = \frac{1}{2}$

Since, the required line is parallel to QR, therefore slope of required line is $\frac{1}{2}$.

$\therefore$ Equation of line passing through $P(4,-3)$ and having slope $\frac{1}{2}$ is

$$y-(-3) = \frac{1}{2}(x-4)$$

(c) Ms. Roy went to a departmental store and bought the following items. The GST rates and the quantity of each items and market price of each are given below : [4]

S.No.	Items	Price per item in ₹	Quantity	GST rate	Amount
1.	Walnut	650	1	5%	
2.	Potato Chips	50	2	0%	
3.	Coffee	80	2	18%	

Find the:

(i) The total amount of SGST paid.

(ii) The total amount of the bill.

Solution.

SP of Walnut = Rs. 650 SGST = 650 × 2.5% = Rs. 16.25

SP of Potato chips = Rs 50×2 = Rs. 100 SGST = 100 × 0% = Rs 0

SP of Coffee = Rs. 80 × 2 = Rs. 160 SGST = 160 × 9% = Rs 14.4

(i) Total amount of SGST paid = 16.25 + 14.4 = Rs. 30.65

(ii) Total amount of bill = total price + GST = Rs. 910 + 2×Rs.30.65

= Rs. 971.3

Question 11.

(a) Mr. Sharma receives an annual income of Rs 900 in buying Rs 50 shares selling at Rs 80. If the dividend declared is 20%, find the:

(i) Amount invested by Mr. Sharma.

(ii) Percentage return on his investment. [3]

Solution.

Given, dividend = Rs 900, face value = Rs 50 and market value = Rs 80.

Let the number of the share be n.

Then, $50 \times \frac{20}{100} \times n = 900$

$n = \frac{900}{10} = 90$

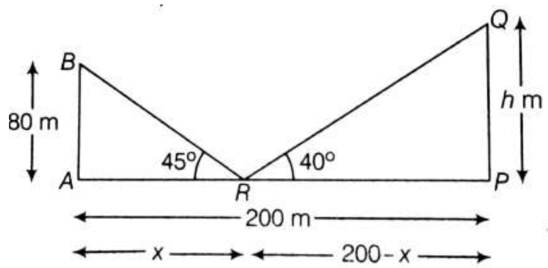
(i) Amount invested by Mr. Sharma = 90 x 80 = Rs 7200.

(ii) Percentage return = $\frac{\text{Income}}{\text{investment}} \times 100\% = \frac{900}{7200} \times 100\% = 12.5\%$

(b) Two poles AB and PQ are standing opposite each other on either side of a road 200 m wide. From a point R between them on the road, the angles of elevation of the top of the poles AB and PQ are 45° and 40° respectively. If height of AB = 80 m, find the height of PQ correct to the nearest metre. [3]

Solution.

We have , AB and PQ as two poles.



Let PQ = h m and AR = x m.

So, PR = (200 - x) m

Now, in triangle ABR,

$$\tan 45^\circ = \frac{AB}{AR}$$

$$1 = \frac{80}{x} \quad => \quad x = 80m$$

∴ PR = 200 - x = 200 - 80 = 120 m

Now, in the triangle PQR,

$$\tan 40^\circ = \frac{PQ}{PR}$$

=> $0.84 = \frac{PQ}{120}$

=> PQ = 0.84 x 120 = 101 m.

(c) Construct a triangle PQR, given RQ = 10 cm, ∠PRQ = 75° and base RP = 8 cm. Find by construction:

The locus of points which are equidistant from QR and QP.

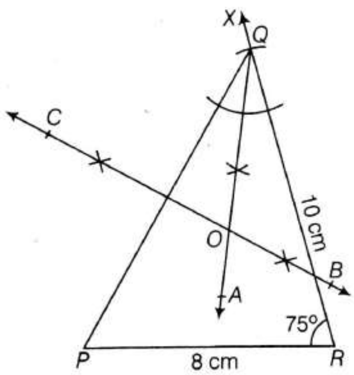
The locus of points which are equidistant from P and Q.

(iv) Mark the point O which satisfies conditions (i) and (ii). [4]

Solution.

Steps of construction

- Draw a line segment *RP* of length 8 cm.
- At *R*, draw a ray *RX* making an angle of 75° with *PR*.
- Draw an arc *RQ* of length 10 cm.
- Join *P* to *Q* to form Δ *PQR*.



(i) Draw the angle bisector of ∠*PQR* and name it *QA*. *QA* is the required locus of points which are equidistant from QR and *QP*.

(ii) Draw the perpendicular bisector of *PQ* and name it *BC*. *BC* is the required locus of points which are equidistant from *P* and *Q*.